Q) The positive rational numbers may be arranged in the form of a simple series as follows
(1 / 1),
(2 / 1), (1 / 2),
(3 / 1), (2 / 2), (1 / 3),
(4 / 1) , (3 / 2), (2 / 3), (1 / 4), ...

Show that (p / q) is the [{(1 / 2) (p + q - 1) (p + q - 2)} + q]th term of the series.

Answer:

As is visible, this relates to the partitioning of numbers into two parts without repetition and with order being sensitive.
Let p+q = t.
Let us see the partitioning of 't'.
For
1st term
't' =
2 = 1+1.

2nd term
3 = 2+1
= 1+2. And so on.

As one can see each 't' can be partitioned in 't-1' ways.
So, till the end of the group where sum of numerator and denominator = 't-1' there would be total of
1/2 * (t-2)(t-1) terms.
Further the 1st term of every group sum giving t = p+q starts with q = 1.
Thus, the (p/q) = would be at [1/2 * (t-2)(t-1) + q] position. Where t = p+q.
Note:-This is the way in which Cantor arranged rational numbers and proved that rational numbers are countably infinite.